Solving VRP in an Indian Transportation Firm through Clark and Wright Algorithm: A Case Study

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Abstract – This article represents a case of an Indian transportation firm wherein vehicle routing problem has been solved using Clark and Wright algorithm. Vehicle routing problems have generally an exact solution, heuristic based solution or meta-heuristic based solution. Clark and Wright method basically involves a heuristic based approach. This article suggests the possible improvements in loading of the vehicles while optimizing the route to enhance profits.

Index Terms – VRP, Route Optimization, Clark and Wright Method.

1. INTRODUCTION

The Vehicle Routing Problem (VRP) is a set of problems that deals with how the goods are distributed between various depots to and from the customers. [1].

Dantzig and Ramser 1959 described the goal of VRP to design route(s) for vehicles stationed at a depot(s) in order to deliver the goods to the customers in minimum time, distance and cost.

Several variants and specializations of the vehicle routing problem exist. The basic variation or the classical VRP is Capacitated Vehicle Routing Problem (CVRP).

In CVRP, the demand is not variable and is known in advance. There is also a capacity constraint on the vehicles. The objective of CVRP is to minimize the cost of transportation by varying other functions such as length of routes, number of vehicles or travel time. (Choosak Pornsing, 2014)[2].

Other variants of VRP are:

1.1 VRP with Time Windows

VRPTD is similar to capacitated VRP with an added constraint of a time window in which the arrival of the goods to the customer are allowed only in a defined interval. Moreover, there is a time window at the depot, which ensures that all the routes begin and end within the time interval defined for that depot.

1.2 VRP with Pick-up and Delivery

Here the provision of a customer returning some products is included. All delivery demands start from the depot like in the basic VRP and all pick-up demand should be returned to the depot[3]

No interchanges of products between customers are allowed. Using one depot, this procedure is called single sourcing VRPPD. An important restriction of the VRPPD is that the vehicle capacity may never be exceeded.

1.3 Multiple Depot VRP

It is a VRP in which there are multiple depots through which service can be provided to the customers. Every depot has a fleet of vehicles and which returns to its respective depot.

1.4 Periodic Vehicle Routing Problem

The PVRP focusses on minimizing the vehicle fleet and the time needed to supply goods to all the customers [3]. In periodic VRP the time period decided the transporter, which may be one day, one week, or one month.

1.5 Stochastic VRP

The SVRP differs from the basic VRP by the introduction of some elements of variability, like random travel times, service times and demands. These elements are stochastic and one makes assumptions on the probability distribution of these elements

1.6 Split Delivery VRP

In the Classical VRP each vehicle visits only one customer and no split delivery is allowed but in SPVRP all these restrictions are removed and split delivery is permitted.(Claudia Archetti, A. Hertz ,et al).[4]

1.7 VRP with Back hauls

In VRPBH the commodities can be demanded or even returned by the customer. Also first, all the deliveries are made on every route before any pickup of a commodity. [5]

This dissertation considers the Vehicle Routing Problem with pick and delivery. The objective of the problem is minimizing the tour length, minimizing the transportation cost and minimizing the number of routes or vehicles.

2. RELATED WORK

The various methods used for solving the VRP are discussed in this review. Depending upon the constraints, objective and computational data available for solving a given problem VRP solution methods are classified into 3 categories:

2.1. Exact Approach

Exact algorithms require too much computation time but they guarantee to obtain the optimum solution. Exact approach is generally of two types:

2.1.1. Branch and Bound Method

In Branch and Bound method all the possible solutions of a VRP are searched and of these the promising solutions are obtained. All the solutions that appear to be non-optimal are discarded. After obtaining the solutions lower and upper bound limits are set as constraints that result in optimal solution. (Chao Lu, Lei-shan Zhou et al.)[6]

2.1.2. Branch and Cut Method

The Branch and Cut method is a composite of the branch and bound method and the cutting plane method. The cutting plane method adds linear inequalities, called cuts, to the problem in order to define an as small as possible feasible set of the objective values. (Lysgaard et al., 2004). [7]

2.2. Heuristics

Unlike the Exact approach a Heuristic does not result in an exact optimal solution but near optimal solutions. The biggest advantage of Heuristicsis their lesser computation time as compared to exact approach and still providing a near good solution.

Broadly, classifying Heuristics are of two types:

2.2.1. Tour Construction Heuristics

They involve construction of a tour from scratch following some construction criteria and stop whenever an initial tour is formed. They involve following approaches:

2.2.1.1. Nearest Neighbor Algorithm

In the Nearest Neighbor procedure (G. Gutin, A. Yeo et al, 2002)[8], the salesman starts from its depot then according to the distance matrix , the customer nearest to depot is visited and from there the next nearest customer is visited and so on. The algorithm stops when all customers are on the tour.

2.2.1.2. Insertion Algorithm:

Insertion heuristic starts with a tour on small subsets and then further extends the tour by insertion of the remaining nodes (Rosenkrantz et al., 1977). In this algorithm the cost of insertions minimized by inserting the node between two consecutive nodes in the selected tour.

2.2.1.3. Clark and Wright Algorithm:

In Clark and Wright method first a distance matrix is prepared for all the vehicles that illustrates the distance between customers and depot and also between the customers (Cordeau et al., 2005) [9]. Then savings cost for each vehicle is computed. Until the capacity constraint is violated the vehicles with maximum savings cost are merged to same vehicle. Based on the values of these savings, the customers are sequentially joined into routes starting with the customer combination yielding the largest cost savings until no further savings can be achieved.

2.2.2. Tour Improvement Heuristics

These algorithms search for the best tour among a neighborhood of the given feasible tour. This neighborhood depends on the tour modification procedure. They involve the following approaches:

2.2.2.1. K-Opt Heuristic

This approach removes k arcs and reconnects the routes in another possible way to find a better solution.(Cordone and Wolfler-Calvo 1997) .If a lower objective function is found, the new route configuration is accepted. In this heuristic a set of initial solutions are created and the best one among them is selected. The procedure stops if no further improvements can be obtained.

2.2.2.2. 2- OPT Procedure:

2-opt Procedure removes two arcs from the initial tour and replaces two different arcs that improve the quality of the tour. The new arcs are chosen so that the new solution is still a tour. (CROES, G.A., 1958). When such a modification is done, the new tour is treated as the initial tour and the modifications are seemed on this new solution. Algorithm terminates when there is no possible improvement. Improvement heuristics getting stuck in local optima has been their major drawback

2.3. Metaheuristics

Metaheuristics are more generic solution schemes that combine several heuristic methods. The drawback of Heuristics is bypassed by this approach. The solutions obtained by this method has the potential to be of higher quality than that obtained by only one heuristic method. The various approaches in Metaheuristics are:

2.3.1. Tabu Search

Tabu search (Glover 1989) is a metaheuristic with focus on the iterated local search procedure. In TS a Tabu List is prepared based on short term memory that that forbids certain possible solutions.TS also allows bad solutions to escape from local minima. Short term cycling is avoided in TS by storing the recently visited solutions. (Toth and Vigo et al.) [10]

2.3.2. Simulated Annealing:

Annealing is a process in which the metal is heated above its critical temperature and then cooled slowly to enhance the microstructure of a metal. The most crucial or critical factor here is the cooling rate as is cooling is done faster it results in reduction in crystalline structure of the metal.SA follows this annealing processing SA for every iteration from the current solutions a neighborhood solution is selected randomly and if this selected solution optimizes the problem it is selected. (Scott Kirkpatrick, C.Danielet al.) [11]

2.3.3. Ant Colony Optimization (ACO):

ACO is a metaheuristic with focus on the route construction. ACO works on the Ant framework. Most ants have no visual ability but still manage to find the shortest roué from their nest to the food. Ants communicate among themselves using a chemical compound known as pheromone.ACO uses artificial ants for optimization process.ACO is implemented by associating pheromone weights with each arc. Every artificial ant uses a positive feedback mechanism to reinforce these arcs belonging to a good solution. (Rizzoli et al., 2007) [12]

2.3.4. Genetic Algorithm (GA):

Genetic Algorithm is a nontraditional optimization technique. GA is basically a search algorithm based on the mechanics of genetic and natural selection. GA works on the process of recombination. This process combines several individuals from the parent population to create new individuals. To create one new individual mostly two parents are selected and their most desirable features are combined in order to create an offspring solution, i.e., the new individual. After the new population is generated, the third process starts the mutation. Each individual is subjected to mutation, which may allow for escaping local minima by preventing the population of chromosomes from becoming too similar to each other.

The result after the first three processes is the final offspring population. The fourth and final process of GA concerns the selection of a new starting population and saving the current best solution. Every offspring individual is rated by one or more criteria and the best or most promising individuals are promoted to a new parent population.

Out of all the algorithms mentioned above, Clark and Wright algorithm is used to solve the given problem. Reason being that Clark and Wright algorithm does not require huge computational data, involves simple calculations that decreases the chances of error in the optimal solution. In addition, Clark and Wright Algorithm is considered the best method to solve the VRP in cases where goods are delivered to customers from a depot and a number of vehicles are available for transport.

3. CASE COMPANY

The Case company (logistics firm) deals in logistics services including warehousing services, logistics solutions, freight forwarding. The Company has a warehouse in Delhi and supplies materials to five customers each at different locations i.e. Gurgaon, Jalandhar, Panipat Ghaziabad, Mathura. The firm books the consignment from its clients and issues them the consignment note. The goods are either delivered to the firm at its booking office but many times the firm has to pick up the goods from the factory premises of its clients.

3.1. Present Scenario

The firm has a fleet of 9 vehicles each of 3-ton capacity but is only able to cater to 30% of the logistics demand and has to outsource the further demands to other transportation firms. The firm transports goods on Monday, Wednesday and Friday every week.

3.2. Problems faced by Company

- Presently the company is using one vehicle to one demand point. The vehicles are loaded at the warehouse and dispatched to the customer, where the delivery goods are unloaded and pick-up goods are loaded. After loading the goods, vehicles are back to warehouse. In this way, single customer's demand is fulfilled by single vehicle.
- As a result of above process the tour length of each vehicle is doubled and number of vehicle is also increasing that makes the current system florid.
- Also due to using single vehicle for single customer, the transportation cost of the company is increase and the profit of the firm is decreases.
- The firm is completely manually operated so chances of errors are more.
- Due to increase in tour length, the transportation time is also increases. So, client inconvenience is increased.

The details of customers and their capacity is shown in the table:

Day	Warehouse	Total Tour Distance	Weight unloaded (ton)	Weight Loaded (ton)	Unloaded at
Monday	Delhi	84	2	1	Gurgaon
	Delhi	720	1	2	Jalandhar
	Delhi	176	2	1	Panipat

	Delhi	80	2	1	Ghaziabad
	Delhi	360	2	2	Mathura
TOTAL DISTANCE : 1420 KM					
Wednesday	Delhi	1440	6	5	Jalandhar
	Delhi	176	3	3	Panipat
	Delhi	720	6	4	Mathura
TOTAL DISTANCE : 2336 KM					
Friday	Delhi	252	9	7	Gurgaon
	Delhi	720	3	2	Jalandhar
	Delhi	352	6	4	Panipat
	Delhi	80	3	2	Ghaziabad
	Delhi	360	2	1	Mathura
TOTAL DISTANCE: 1764 KM					

Table 1: CUSTOMER DETAILS AND THEIR CAPACITY

4. METHODOLOGY

The methodology involved in addressing the above problem using Clark and Wright saving heuristics is as follows:

- Identify Distance Matrix
- Identify Savings Matrix
- Rank Savings
- Assign Customers to vehicles
- Sequence Customers

4.1. Identity Distance Matrix

A distance matrix is prepared, in which the distance between warehouse and customers and the distance between Customers in entered.

DISTANCE MATRIX						
	Warehouse	Customer 1	Customer 2	Customer3	…	Customer n
Warehouse	0	d(w,1)	d(w,2)	d(w,3)	…	d(w,n)
Customer 1		0	d(1,2)	d(1,3)	…	d(1,n)
Customer 2			0	d(2,3)	…	d(2,n)
Customer 3				0	…	d(3,n)
⋮						⋮
Customer n						0

Table 2: DISTANCE MATRIX

4.2. Identify Savings Matrix

Using the distance matrix, savings for all pairs of customers are calculated. SavingsS (i, j) represents the saving in traveling distance obtained by assigning customer i and j to same vehicle instead of assigning them to two different vehicles.

$$s(i,j) = d(i,w) + d(j,w) - d(i,j)$$

The savings are then arranged in the matrix:

Table 3: SAVINGS MATRIX

Savings Matrix					
	Customer 1	Customer 2	Customer 3	…	Customer n
Customer 1	0	s(1,2)	s(1,3)	…	s(1,n)
Customer 2		0	s(2,3)	…	s(2,n)
Customer 3			0	…	s(3,n)
. :					: .
Customer n					0

4.3. Rank Savings

Now the savings are ranked and listed in the deceasing order of their magnitude. The pair of customers with the highest savings are merged to the same vehicle.

4.4. Assign Customers to vehicles

In this step, customers are merged. Before merging the pair, availability of capacity is checked. On availability of the capacity the pair of customers with the maximum savings are merged in same vehicle. If the capacity is not available in the vehicle then the next pair is merged in the new vehicle.

Then again check the availability, if capacity is not available then not merge them in same vehicle. Now, the next pair are merged and assigned to a new vehicle based on availability of capacity.

4.5. Sequence Customers

In this step sequencing of the customers merged to the same vehicle is done. For every route the distance covered depends on the sequencing of customers.

We use Nearest Neighbor Algorithm for sequencing. In sequencing first, consider the customers who are assigned to the same vehicle. The relevant distances for sequencing are received from the distance matrix. Now, select among the customer who are assigned the same vehicle, which is the nearest to warehouse. Therefore, the vehicle will first travel from warehouse to nearest customer. Then select then next customer having nearest to the customer first. In similar manner, the route is decided for all vehicles.

5. ASSUMPTIONS

The major assumptions to simplify and analyze the data are provided below: -

1. Each route will start from and end at depot. Each vehicle will leave the depot, arrive at determined customers, and arrive back to the depot.
2. Demands at each stop (both delivery and pickup goods) are known.
3. The demand at each stop cannot be divided or split.
4. Travel time between each stop are known and with accuracy.
5. Pick up are performed after a delivery of the goods.
6. Pickup and delivery are made by the same vehicle.
7. Unloading time per stop is constant or every stop.
8. Instead of 3-ton capacity vehicles used earlier, the new proposed is 9-ton capacity vehicles.

6. PROBLEM SOLUTION

	Ware-house	Gurgaon	Jalandhar	Panipat	Ghaziabad	Mathura
Warehouse	0	42	360	088	40	180
Gurgaon		0	400	122	68	160
Jalandhar			0	290	395	535
Panipat				0	115	240
Ghaziabad					0	160
Mathura						0

Table 4: DISTANCE MATRIX

STEP 1: Identifying Distance Matrix

STEP 2: Identifying Savings Matrix

Saving Formula:

$$s(i,j) = d(i,w) + d(j,w) - d(i,j)$$

S (1,2) =2 s (2,3) = 158

s (1,3) =8 s (2,4) = 5

s (1,4) =14 s (2,5) = 5

s (1,5) =62 s (3,4) = 13

s (4,5) = 60 s (3,5) = 28

	Gurgaon	Jalandhar	Panipat	Ghaziabad	Mathura
Gurgaon	0	2	8	14	62
Jalandhar		0	158	5	5
Panipat			0	13	28
Ghaziabad				0	60
Mathura					0

Table 5: SAVINGS MATRIX

Step 3: Rank Savings

Savings calculated are now arranged in decreasing order:

Ranking list:

(2,3) (1,5) (4,5) (3,5) (1,4) (3,4) (1,3) (2,4) (2,5) (2,4) (2,5) (1,2)

Day- Monday

Step 4: Assign the customer to vehicles:

All the customers are merged to one truck only

Step 5: Sequence of the customer

Warehouse - 4 - 1- 3 - 5 - 2 - Warehouse

Total tour length- 1185 Km

Day - Wednesday

Assign the customer to vehicles

Customer 2 and 3 are managed by 1 vehicle

Customer 5 is managed by 1 vehicle

Sequence of the customer

Warehouse - 2 - 3 - Warehouse

Warehouse - 5 - Warehouse

Total tour length- 1098 Km

Day - Friday

Assign the customer to vehicles

Customer 2 and 3 are managed by 1 vehicle

Customer 4 and 5 are managed by 1 vehicle

Customer 1 is managed by 1 vehicle.

Sequence of the customer

Warehouse - 2 - 3 - Warehouse

Warehouse - 1 - Warehouse

Warehouse - 4 - 5 - Warehouse

Total tour length- 1166 Km

7. RESULTS AND DISCUSSIONS

7.1. SAVINGS

7.1.1. Without using Clark and Wright Algorithm

Total distance covered on Monday - 1420 Km

Total distance covered on Wednesday - 2336 Km

Total distance covered on Friday - 1764 Km

Total Distance in a week - 5520 Km

Cost of fuel/liter in a 3-ton capacity truck - Rs 9/liter

Total cost in a week - Rs 49680

7.1.2. Using Clark and Wright Algorithm

Total distance covered on Monday - 1185 Km

Total distance covered on Wednesday -1098 Km

Total distance covered on Friday -1166 Km

Total distance in a week -3449 Km

Cost of fuel/litre in a 9 ton capacity truck - Rs 10/litre

Total cost in a week - Rs 34490

Net savings in a week - Rs 15190.

7.2. COST COMPARISON

Table 6: COST COMPARISON TABLE

	Without Applying Clark and Wright's Algorithm	After Applying Clark and Wright's Algorithm
Total Tour Length	5520 Km	3449 Km
Total Cost of Transportation	Rs 49680	Rs 34490

8. CONCLUSION

The usual objective in vehicle routing problems is to minimize the total distance travelled. However, in several real life applications other objectives are also important. This could be minimizing the total number of vehicles used, equalizing the load of tours, or minimizing the length of the longest route.

In this thesis, the author presented a case of a transportation firm and analyzed the total cost with and without using the Clark and Wright's algorithm.

The comparison gave a savings of 30.57% on total cost in a week by using the Heuristic of Clark and Wright. In addition, the number of vehicles used in transportation have also been reduced by a significant amount.

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